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considered of first importance, then the question of the historicity of this content becomes clear and constantly more important. I would like very much to learn your opinion about the hypotheses of Benjamin W. Smith (*Der vorchristliche Jesus*) and Thomas Whittaker (*Origins of Christianity*). It would be serviceable if you would bring these radical theories into contrast with my comparatively conservative view, and would instruct your public about this crisis at the very beginning of primitive Christianity.

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THE LOGICAL ASPECT OF THE THEORY OF HYPER-SPACES.

A study of the articles on hyper-spaces by Cassius J. Keyser in *The Monist* for January 1906 and by W. B. Pitkin in the January number this year seems to me to reveal pretty clearly an error into which certain mathematicians have fallen in the consideration of this subject. But the curious thing in Mr. Pitkin's article is, that, although he is sufficiently aware of the error to perceive it in others, he partly falls into it himself in pointing it out in them. In short, he only partly perceives the error.

The error is in trying to generalize the idea of dimensionality. The truth is that dimensionality is a property peculiar to space. Dimensions determine space and other properties determine other things. But, as Mr. Pitkin says, while the dimensions of space are determinants, the determinants of things other than space cannot be called dimensions.

But the proof of this fact depends, I think, on differences more thorough and radical than Mr. Pitkin is aware of.

The striking peculiarity of the dimensions of space which distinguishes them from the determinants of sound or color or anything else, is the fact that they are all identically the same in kind. For convenience in conversation we refer to them as length, breadth, and height. But, as regards the determination of a given space, it is of absolutely no importance which dimension we term length, which breadth, and which height. A striking result of this peculiarity is that any two or all three of these dimensions may be multiplied together and an intelligible result produced thereby. The fact is that the dimensions of space are themselves space-forms of space.

The three dimensions of a space are all lines. We use one-dimensional space to express the dimensions of three-dimensional space. Besides this, it is not altogether accurate to describe space, as we know it, as three-dimensional. It is not the whole truth. For we are just as familiar with two-dimensional and one-dimensional space as we are with three-dimensional—to say nothing of the point which is of no dimensions. Three is simply the greatest number of dimensions that most men are capable of perceiving. And it is just this actual existence of space of less than three dimensions which opens up the possibility of the existence of space of more than three dimensions.

Now what is the case as regards the determinants of a musical note—pitch, timbre, and loudness? Are these all identically the same in kind? Could we just as well call pitch loudness, or timbre pitch? Moreover is there any such thing as a one-dimensional tone—say of loudness only, or pitch only? or a two-dimensional tone of pitch and timbre without loudness? or can we multiply pitch and timbre together and produce a result which has any meaning? or can we obtain anything intelligible by multiplying pitch, timbre and loudness together? All this is sheer nonsense! As well talk of multiplying coal-tar by sawdust and thus producing music.

These considerations completely knock the bottom out of the argument by which, from the analogy of tones, Mr. Pitkin seeks to establish the impossibility of a four-dimensional space. Nothing at all analogous is to be found in the science of sound or color or anything else. The three known dimensions of space being all exactly the same kind, the existence of a fourth dimension of the same kind is easily conceivable. But in the case of a musical note, if we think of a fourth determinant, the question at once arises, of what kind is it to be? In the case of a fourth dimension of space, no such question arises. A tone having three determinants of three different kinds, plus a fourth of the same kind as one of the other three, is a grotesque lop-sided conception possessing no sort of resemblance to a space having an extra dimension of the same kind as the three which we are now able to perceive. Moreover, a precisely similar argument might be used to disprove the existence of space of *less* than three dimensions; whereas it is a matter of common every-day experience to all men that such spaces actually exist.

These considerations seem to me so obvious that I am surprised that it should be necessary to point them out. I am no scientist. I am a mere man-in-the-street, or rather—in the jungle. I am, however, an engineer by profession; and perhaps, as such, I have had more to do in the way of measuring spaces than falls to the lot of men in other professions. I have been at it every day of my life for the last twenty and odd years. And it is possibly this extreme familiarity with the dimensions of space that has given me a more than usually vivid conception of what dimensionality really is. Moreover this is not really a question of science at all, but of philosophy—a question of pure reason—of logic—of mathematics. And that is a faculty born in man, and one in which those most renowned in scientific research are often deficient. The discovery of facts and the discovery of the necessary relationships between facts are two distinct departments. The philosopher does not concern himself with the discovery of facts. He is content to get his facts ready made from the scientist. And it would perhaps be well if scientists would leave philosophy to philosophers. In science, the rôle of the philosopher is that of the spectator, who sees most of the game.

The logical aspect of the theory of hyper-spaces is perfectly sound. Philosophy has shown conclusively that four-dimensional space *may* exist. The acceptance of the idea introduces no discord, no break in the continuity of our knowledge. It is not ***contradicted*** by any known scientific facts. *But*—whether four-dimensional space actually has any real existence has yet to be proved. That is a question of simple scientific fact. The claim of some that they have by practice actually become capable of intuiting a fourth dimension is no proof of its existence. It only proves that these people have very strong and vivid imaginations. By mathematics one might calculate to the minutest detail what a four-dimensional body would be like. And by an adequate imagination we might picture it to ourselves in the minutest detail. The novelist, Mr. Wells, has carried out a process very similar to this with regard to the hypothetic inhabitants of the moon. But scientific proof of the existence of the inhabitants of the moon, and of the fourth dimension, are at present both conspicuous by their absence.

The lines which such scientific proof must take may perhaps be roughly indicated in advance.

It must be either direct or indirect.

The direct proof would be to make the fourth dimension actually visible to us by some kind of microscope or other-scope.

The indirect way would be as follows. If there is a fourth dimension, then certain results must follow which can be demonstrated in the three-dimensional world—then demonstrate them. For instance, if a space be completely closed as regards its three apparent dimensions, but if nevertheless there is an unapparent fourth dimension, in the direction of which it is wide open, then a solid body could be introduced into that space along that fourth dimension. Search for an instance of such a phenomenon! Some spiritualists claim to have cases on record.

Indirect evidence, however, is not conclusive, unless it can be shown that no other explanation is possible. And, trickery apart, explanations other than a fourth dimension, but perhaps no less marvelous, would, in this case, be conceivable.

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A MULTI-DIMENSIONAL SPACE CONCEPTION.

(A Translation.)

To the Editor of The Monist:

First of all I must express my thanks to you for the unusually sympathetic words with which you commented upon my new geometrical theory in the April number of *The Monist* (p. 316). For this reason I feel the more in duty bound to call your attention to one place in the review which is inaccurate because of my own fault.

After you placed an emphasis, quite rightly, upon the fact that my theory "beats Riemann's curved space and also Bolyai and Lobachevsky," you add that "even the believers in the fourth dimension must confess that they are left behind." This is evidently founded upon the oft repeated statement in my book that extended space can have not only four but even five and six dimensions but no more. But in a correction which I have added as an additional independent supplement I have corrected this statement to the effect that the supposed five- and six-dimensional octahedral space would have to be identical with the quadratic five- and six-dimensional space; and that since this last is not possible, extended space can not have more than four dimensions.

Nothing indeed has been gained *in principle* by this correction, for in principle only the assertion which is carried out in the book is correct, namely that *extended* space can not exceed a definite